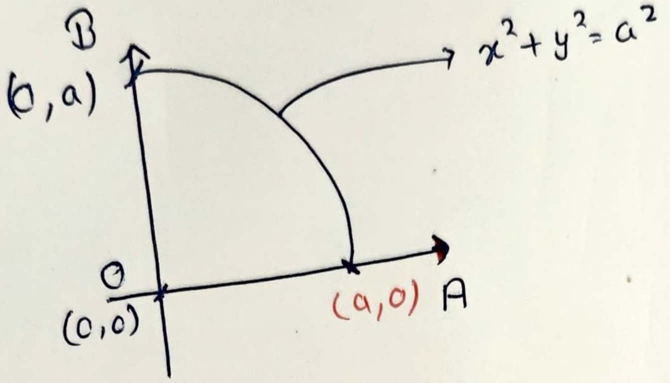
Q.B) Greens theorem.

Q8) verify greens theorem for $\overline{F} = x\hat{i} + y^2\hat{j}$ over the first quadrant of the circle $x^2+y^2=a^2$.



L.H.S = R.H.S

$$\oint_C x \cdot dx + y^2 \cdot dy = \iint_R \left(\frac{\partial y^2}{\partial x} - \frac{\partial x}{y} \right) \cdot dx \cdot dy$$

- L.H.S

$OABO$

$$W = \int_{OABO} \overline{F} \cdot d\overline{r}$$

$$= \underbrace{\int_{OA}}_{W_1} + \underbrace{\int_{AB}}_{W_2} + \underbrace{\int_{BA}}_{W_3} \quad \text{—(I)}$$

$$W_1 = \int x \cdot dx + y^2 \cdot dy$$

$y=0 \quad x = 0 \to a$

$dy = 0$

$$W_1 = \int_0^a x \cdot dx$$

$$\boxed{W_1 = \frac{x^2}{2} \Rightarrow \frac{a^2}{2}} \text{—(II)}$$

$$W_2 = \int x \cdot dx + y^2 \cdot dy$$

$x = a\cos\theta, \; y = a\sin\theta$

$\boxed{dx = a(-\sin\theta) \cdot d\theta}$

$\boxed{dy = a\cos\theta \cdot d\theta}$

$$W_2 = \int_0^{\pi/2} a\cos\theta \cdot a(-\sin\theta) \cdot d\theta + a^2\sin^2\theta \cdot a\cos\theta \cdot d\theta$$

$$W_2 = -a^2 \int_0^{\pi/2} \cos\theta \cdot \sin\theta \cdot d\theta + a^3 \int_0^{\pi/2} \sin^2\theta \cdot \cos\theta \cdot d\theta$$

$$w_2 = -a^2\int_0^{\pi/2} t\cdot dt + a^3\int_0^{\pi/2} t^2\cdot dt$$

..... Put $\boxed{\sin\theta = t}$

$\boxed{dt = \cos\theta\cdot d\theta}$

$$w_2 = -\frac{a^2}{2}\left[t^2\right]_0^{\pi/2} + \frac{a^3}{3}\left[t^3\right]_0^{\pi/2}$$

$$\boxed{w_2 = -\frac{a^2}{2} + \frac{a^3}{3}} \quad -(iii)$$

$$w_3 = \int x\cdot dx + y^2\cdot dy$$

$x = 0 \quad y = a \to 0$

$$= \int_a^0 0\cdot dx + y^2\cdot dy$$

$$= \left(\frac{y^3}{3}\right)_a^0$$

$$w_3 = \frac{0}{3} - \frac{a^3}{3}$$

$$\boxed{w_3 = -\frac{a^3}{3}} \quad -(iv)$$

from (i) – (ii) – (iii) – (iv)

$$W = \frac{a^2}{2} + \left(-\frac{a^2}{2} + \frac{a^3}{3}\right) - \frac{a^3}{3}$$

$$\boxed{W = 0}$$

R.H.S

$$= \iint \left(\frac{\partial y^2}{\partial x} - \frac{\partial x}{\partial y} \right) \cdot dx \cdot dy$$

$$\Rightarrow \boxed{\frac{\partial y^2}{\partial x} = 0 \;, \; \frac{\partial x}{\partial y} = 0}$$

$$= \iint_R 0 \cdot dx \cdot dy = 0$$

$$\boxed{\text{L.H.S} = \text{R.H.S}}$$

Hence proved.