Q7) Find the work done by the force $(x^2-yz)\hat{i}+(y^2-xz)\hat{j}+(z^2-xy)\hat{k}$ in taking particle from $(1,1,1)$ to $(2,2,0)$.

Solⁿ Here, The path is not mentioned.

$$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2-yz & y^2-xz & z^2-xy \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}(z^2-xy) - \frac{\partial}{\partial z}(y^2-xz)\right] - \hat{j}\left[\frac{\partial}{\partial z}(x^2-yz) - \frac{\partial}{\partial x}(z^2-xy)\right]$$

$$\hat{k}\left[\frac{\partial}{\partial x}(y^2-xz) - \frac{\partial}{\partial y}(x^2-yz)\right]$$

$$= \hat{i}\left[-x+x\right] - \hat{j}\left[-y+y\right] + \hat{k}\left[-z+z\right]$$

$\boxed{\nabla \times \bar{F} = 0}$ → curl of F is 0.

$$W = \int \bar{F} \cdot d\bar{r}$$

$$= \int_{\substack{z=c \\ y=c}} (x^2-yz) \cdot dx + \int_{\substack{x=0 \\ z=c}} (y^2-xz)\, dy + \int_{\substack{x=0 \\ y=0}} (z^2-xy) \cdot dz$$

$$= \left[\frac{x^3}{3} - xyz + \frac{y^3}{3} + \frac{z^3}{3} \right]_{(1,1,1)}^{(2,2,0)}$$

$$= \left[\frac{2^3}{3} - 0 + \frac{2^3}{3} + 0 \right] - \left[\frac{1}{3} - 1 + \frac{1}{3} + \frac{1}{3} \right]$$

$$= \left[\frac{8}{3} + \frac{8}{3} \right] - \left[1 - 1 \right]$$

$$\boxed{w = \frac{16}{3}}$$