

Q6). Find the work done in moving a particle from $(0, 1, -1)$ to $(\pi/2, -1, 2)$ in a force of field $F = (y^2 \cos x + z^3) \mathbf{i} + (2y \sin x - 4) \mathbf{j} + (3xz^2 + 2) \mathbf{k}$.

Solⁿ. Here in question, there is no mention of Path, hence, we need to find the curl of F , if curl of F is zero then we can choose a path by own.

$$\nabla \times F = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 \cos x + z^3 & 2y \sin x - 4 & 3xz^2 + 2 \end{vmatrix}$$

$$= \left[\frac{\partial}{\partial y} (3xz^2 + 2) - \frac{\partial}{\partial z} (2y \sin x - 4) \right] \mathbf{i}$$

$$- \mathbf{j} \left[\frac{\partial}{\partial x} (3xz^2 + 2) - \frac{\partial}{\partial z} (y^2 \cos x + z^3) \right]$$

$$+ \mathbf{k} \left[\frac{\partial}{\partial x} (2y \sin x - 4) - \frac{\partial}{\partial y} (y^2 \cos x + z^3) \right]$$

$$= (0 - 0) \mathbf{i} - \mathbf{j} [3z^2 - 3z^2] + \mathbf{k} [2y(\cos x) - 2y \cos x]$$

$$= \underline{\underline{0}}$$

curl of F is 0.

if the F is irrotational then integral is exact difference

$$= \left[\int_{y, z \text{ const}} (y^2 \cos x + z^3) \cdot dx + \int_{x=0, z=\text{const}} (2yz \sin x - 4) \cdot dy + \int_{x=0, y=0} (3xz^2 + 2) \cdot dz \right]_{(0, 1, -1)}^{(\pi/2, -1, 2)}$$

$$= \left[(y^2 \sin x + xz^3) - 4y + 2z \right]_{(0, 1, -1)}^{(\pi/2, -1, 2)}$$

$$= \left[(-1)^2 \sin \pi/2 + \frac{\pi}{2} (2)^3 - 4(-1) + 2 \times 2 \right] - \left[(1)^2 \sin 0 - 0(-1)^3 - 4(1) + 2(-1) \right]$$

$$= [1 + 4\pi + 4 + 4 - 0 - 0 + 4 + 2]$$

$$= \underline{\underline{15 + 4\pi}}$$