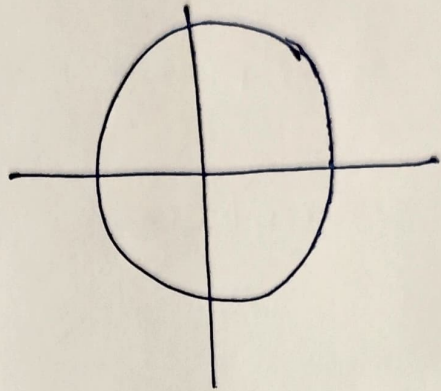


(Q4) Find the work done in moving particle once around the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1, z=0$ under the field of force

$$F = (2x - y + z)\mathbf{i} + (x + y - z^2)\mathbf{j} + (3z - 2y + 4x)\mathbf{k}$$

Solⁿ. Here $z=0$
 $\underline{\underline{dz=0}}$



$$W = \int \vec{F} \cdot d\vec{r}$$

$$= \int (2x - y) \cdot dx + (x + y) \cdot dy \quad \text{--- } (z=0, dz=0).$$

--- ①

Put

$$x = 5 \cos \theta$$

$$y = 4 \sin \theta$$

$$\boxed{dx = 5(-\sin \theta) \cdot d\theta} \quad \boxed{dy = 4 \cos \theta \cdot d\theta} \quad \text{--- ②}$$

From ①-②

~~$$W = \int_0^{2\pi} [2 \times 5(-\sin \theta) - 4 \sin \theta] \cdot 5(-\sin \theta) \cdot d\theta + (5 \cos \theta + 4 \sin \theta) \cdot 4 \cos \theta \cdot d\theta$$~~

$$W = \int_0^{2\pi} [2(5 \cos \theta) - 4 \sin \theta] \cdot 5(-\sin \theta) \cdot d\theta + (5 \cos \theta + 4 \sin \theta) \cdot 4 \cos \theta \cdot d\theta$$

$$W = \int_0^{2\pi} [25(2 \sin \theta \cdot \cos \theta) + 20 \sin^2 \theta] \cdot d\theta + [20 \cos^2 \theta + 16 \sin \theta \cdot \cos \theta] \cdot d\theta$$

$$W = \int_0^{2\pi} [-50 \sin \theta \cdot \cos \theta + 20(\sin^2 \theta + \cos^2 \theta) + 16 \sin \theta \cdot \cos \theta] \cdot d\theta$$

$$W = \int_0^{2\pi} (-34 \sin \theta \cdot \cos \theta + 20) \cdot d\theta \quad (\sin^2 \theta + \cos^2 \theta = 1)$$

$$W = \int_0^{2\pi} [-17(2 \sin \theta \cdot \cos \theta) + 20] \cdot d\theta$$

$$\begin{aligned}
 &= \int_0^{2\pi} (\sin 2\theta + 20) \cdot d\theta \\
 &= -17 \int_0^{2\pi} \sin 2\theta \cdot d\theta + 20 \int_0^{2\pi} d\theta \\
 &= -17 \left[-\cos 2\theta + 20\theta \right]_0^{2\pi} \\
 &= -17 [\dots]
 \end{aligned}$$

$$W = \int_0^{2\pi} [-17(\sin 2\theta) + 20] \cdot d\theta \quad \dots \quad \sin 2\theta = 2 \sin \theta \cdot \cos \theta$$

$$W = \int_0^{2\pi} [17 \cos 2\theta]_0^{2\pi} + [20\theta]_0^{2\pi}$$

$$W = 17 [\cos 4\pi - \cos 0] + 20(2\pi - 0)$$

$$W = 17(-1 + 1) + 40\pi$$

$$\boxed{W = 40\pi}$$