Q3) Evaluate $\int \bar{F}\cdot d\bar{r}$ for $\bar{F} = (2x+3)\hat{i} + xy\hat{j} + (yz-x)\hat{k}$ along the following path.

a) The straight lines from (0,0,0) to (0,0,1) then to (0,1,1) and then to (2,1,1).

$$W = \int w_1 + w_2$$

for w_1 $(0,0,0) \longrightarrow (0,0,1)$

$$\frac{x-0}{0-0} = \frac{y-0}{0-0} = \frac{z-0}{1-0}$$

$x = 0t,\ y = 0t,\ z = t$

$dx = 0,\ dy = 0,\ dz = dt$

$$w_1 = \int \cancel{(2x+3)\cdot dx}^{\,0} + \cancel{xy\cdot dy}^{\,0} + (yz-x)\cdot dz$$

$$w_1 = (yz-x)\cdot dt$$

$$\boxed{w_1 = 0}$$

for w_2 $(0,1,1) \longrightarrow (2,1,1)$

$$\frac{x-0}{2-0} = \frac{y-1}{1-1} = \frac{z-1}{1-1}$$

$x = 2t \quad y = 0t \quad z = 0t$

$dx = 2\cdot dt \quad dy = 0 \quad dz = 0$

$$w_2 = \int (2x+3)\cdot dx + xy\cdot dy + (yz-x)\cdot dz$$

$$w_2 = \int (2x+3)\cdot 2\cdot dt + 0 + 0$$

$$w_2 = \int (2(2t)+3)\cdot 2\,dt$$

$$w_2 = \int (4t+3)\,2\cdot dt$$

$$w_2 = \int (8t+6)\cdot dt$$

$$w_2 = \left[8\frac{t^2}{2} + 6t\right]_0^1$$

$$w_2 = \frac{\cancel{8}^{\,4}}{\cancel{2}} + 6$$

$$\boxed{w_2 = 10}$$

$$W = w_1 + w_2$$

$$\boxed{W = 10}$$

Q3)

b) The straight line joining $(0,0,0) \longrightarrow (3,1,1)$.

$$\frac{x-0}{3-0} = \frac{y-0}{1-0} = \frac{z-0}{1-0} = t$$

$x = 3t$, $y = t$, $z = t$

$dx = 3dt$, $dy = dt$, $dz = dt$

$$W = \int \bar{F} \cdot d\bar{r}$$

$$= \int (2x+3)i + xyj + (yz-x)k \cdot (i\,dx + j\cdot dy + k\cdot dz)$$

$$= \int (2x+3)\cdot dx + xy\cdot dy + (yz-x)\cdot dz$$

$$= \int (2(3t)+3)\,3\cdot dt + (3t)(t)\cdot dt + (t\cdot t - 3t)\cdot dt$$

$$= \int (6t+3)\cdot 3\cdot dt + 3t^2\cdot dt + (t^2-3t)\cdot dt$$

$$= \left[18\frac{t^2}{2} + 9t + \frac{3t^3}{3} + \frac{t^3}{3} - \frac{3t^2}{2}\right]_0^1$$

$$= \frac{\cancel{18}^{\,9}}{\cancel{2}} + 9 + 1 + \frac{1}{3} - \frac{3}{2}$$

$$= 19 + \frac{2-9}{6} = \frac{114-7}{6} = \frac{107}{6}.$$

$$= 19 + \left(\frac{-7}{6}\right)$$