#unit 03 [Question Bank - Solution]

A. Line integral / work-Done.

Q1) Evaluate $\int \bar{F} \cdot d\bar{r}$ for $\bar{F} = 3x^2 i + (2xz - y) j + zk$ along the following paths.

a) straight line joining the point $0,0,0 \rightarrow 2,1,3$

$$\frac{x-0}{2-0} = \frac{y-0}{1-0} = \frac{z-0}{3-0} = t \quad \text{---} \textcircled{1} \qquad \frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$x = 2t$, $y = t$, $z = 3t \longrightarrow dx = 2 \cdot dt$, $dy = dt$, $dz = 3 \cdot dt$

$$w = \int F \cdot dr$$

$$= \int (3x^2 i + (2xz - y) j + zk) \cdot (i\,dx + j \cdot dy + k \cdot dz)$$

$$= \int 3x^2 \cdot dx + (2xz - y) \cdot dy + z \cdot dz$$

$$= \int 3(2t)^2 \cdot dx + (2 \cdot 2t \cdot 3t - t) \cdot dt + 3t \cdot 3 \cdot dt$$

$$= \int 24t^2 \cdot dt + (12t^2 - t) \cdot dt + 9t\,dt$$

$$= 24 \cdot \left(\frac{t^3}{3}\right)_0^1 + 12\left(\frac{t^3}{3}\right)_0^1 - \left(\frac{t^2}{2}\right)_0^1 + 9\left(\frac{t^2}{2}\right)_0^1$$

— we get limit from $\textcircled{1}$

$$= 12 + \frac{\cancel{8}^{\,4}}{\cancel{2}}$$

$$= \underline{\underline{16}}.$$

b) along curve $x = 2t^2$, $y = t$, $z = 4t^2 - t$

$dx = 2 \cdot 2t \cdot dt$, $dy = dt$, $dz = 8t \cdot dt - dt$

$dx = 4t \cdot dt$, $dy = dt$, $dz = (8t - 1) \cdot dt$

$$w = \int 3x^2 \cdot dx + (2xz - y) \cdot dy + z \cdot dz$$

$$= 3 \times (2t^2)^2 \cdot 4t \cdot dt + \left[2 \times 2t^2 \cdot (4t^2 - t) - t\right] \cdot dt$$

$$(4t^2 - t) \cdot (8t - 1) \cdot dt$$

$$= 12 \times 4 \times t^4 \times t \times dt + \left[4t^2(4t^2 - t) - t\right] \cdot dt +$$

$$(32t^3 - 4t^2 - 8t^2 + t) \cdot dt$$

$$= 48t^5 \cdot dt + \left[16t^4 - 4t^3 - t\right] \cdot dt + (32t^3 - 4t^2$$

$$- 8t^2 + t) \cdot dt$$

$$= 48t^5 \cdot dt + 16t^4 \cdot dt - \underline{4t^3 \cdot dt} - \cancel{t \cdot dt} \underline{+ 32t^3 \cdot dt}$$

$$\underline{-4t^2 \cdot dt} - \underline{8t^2 \cdot dt} + \cancel{t \cdot dt}$$

$$= 48t^5 \cdot dt + 16t^4 \cdot dt + 28t^3 \cdot dt - 12t^2 \cdot dt$$

$$= 48\left(\frac{t^6}{6}\right)_0^1 + 16\left(\frac{t^5}{5}\right)_0^1 + 28\left(\frac{t^4}{4}\right)_0^1 - 12\left(\frac{t^3}{3}\right)_0^1$$

$$= \frac{\overset{8}{\cancel{48}}}{\cancel{6}} + \frac{16}{5} + \frac{\overset{7}{\cancel{28}}}{\cancel{4}} - \frac{\overset{4}{\cancel{12}}}{\cancel{3}}$$

$$11 + \frac{16}{5} = \frac{71}{5}$$

c) Along the curve defined by $x^2 = 4y$ and $3x^3 = 8z$ from $x=0$ to $x=2$

$$w = \int \bar{F} \cdot d\bar{r}$$

$$= \int 3x^2 \cdot dx + (2xz - y) \cdot dy + z \cdot dz$$

$$= \int 3x^2 \cdot dx + \left(2x \cdot \frac{3x^3}{8} - \frac{x^2}{4}\right) \cdot dy + \frac{3x^3}{8} \cdot dz$$

---- $y = \frac{x^2}{4}$, $z = \frac{3x^3}{8}$ By question.

$y = \frac{x^2}{4}$

$dy = \frac{2}{4} x \cdot dx$

$\boxed{dy = \frac{x}{2} \cdot dx}$

$\boxed{dz = \frac{9x^2}{8} \cdot dx}$

$$= \int 3x^2 \cdot dx + \frac{6x^4}{8} \cdot \frac{x}{2} \cdot dx - \frac{x^2}{4} \cdot \frac{x}{2} \cdot dx + \frac{3x^3}{8} \cdot \frac{9x^2}{8} \cdot dx$$

$$= \int_0^2 3x^2 \cdot dx + \left(-\frac{x^3}{8}\right) \cdot dx + \frac{51}{64} x^5 \cdot dx$$

$$= 8 - \frac{1}{2} + \frac{51}{6}$$

$$= \frac{48 - 3 + 51}{6}$$

$$= \frac{96}{6} = \underline{\underline{16}}.$$