UNIT-1

ELECTRON BALLISTICS AND OPTICS

Introduction:

- **Kinematical Equations:**

$$s = ut + \frac{1}{2}at^2$$

$$v = u + at$$

$$v^2 = u^2 + 2as$$

Here, s is displacement, u is initial velocity, a is acceleration, v is the final velocity

- Electric field between two parallel plates separated by distance d and having potential difference V is given by

$$E = \frac{V}{d}$$

- **Equipotential Surface:**

A region in a space where every point has the same potential. Equipotential surfaces are always perpendicular to electric field lines.

- **Force exerted on a charged particle in electric field is given by**

$$F = qE$$

Here, q is charge on a charged particle, E is electric field.

- **Force exerted on a charged particle in magnetic field is given by**

$$F = q(v \times B)$$

$$F = qvBsin\theta$$

Here, q is charge on a charged particle, v is velocity of charged particle, B is magnetic field and θ is the angle between velocity and magnetic field vector.

Motion of electron in an electric field

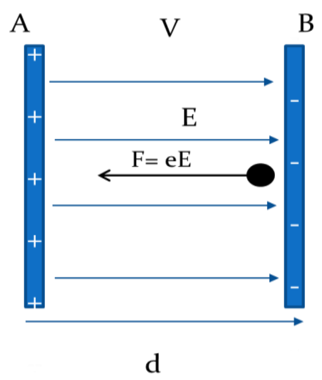
Based on direction of electric field there are two ways of motion of electron through it.

(a) Motion of electron in parallel electric field
(b) Motion of electron in perpendicular electric field

(a) Trajectory (path) of an electron in uniform parallel electric field-

- Let us consider two parallel metal plates A (positively charged) and B (negatively charged).

-Consider an electron moving parallel the electric field.



-When electron enters in the electric field, the force will be opposite to the electric field. So, the electrons accelerate while moving through the electric field and its speed will increase. So the kinetic energy of an electron will change.

-The electron follows the **straight line path.**

Let,

V= potential difference between the plates

d= distance between the plates

Now the force acting on electron is,

$$F = eE \qquad \text{---------------- (1)}$$

By using Newton's second law,

$$F = ma \qquad \text{----------------- (2)}$$

Comparing Eq. (1) and (2)

$$eE = ma$$

$$a = \frac{eE}{m} \qquad \text{-------------- (3)}$$

Let an electron travels a distance 'x' in time 't' and velocity acquired by it is 'v'.

By using kinematical equations we can find the velocity and displacement.

i) The velocity of electron in time 't' is,

$$v = u + at = u + \frac{eE}{m} t \qquad \text{(By using equation 3)}$$

ii) Displacement of an electron in time 't' is,

$$x = ut + \frac{1}{2} at^2 = ut + \frac{1}{2} \frac{eE}{m} \qquad \text{(By using equation 3)}$$

iii) Also, $v^2 = u^2 + 2ax$

$$v^2 = u^2 + 2\frac{eE}{m}x \qquad \text{(By using equation 3)} \text{ ----------------- (4)}$$

So, the final kinetic energy of an electron is,

$$KE = \frac{1}{2}mv^2$$

Put the value of v^2 from eq. (4)

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + \frac{1}{2}m(2\frac{eE}{m}x)$$

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + eEx$$

If we assume initial velocity of electron is zero then, u = 0

$$\frac{1}{2}mv^2 = eEx$$

Here, $Ex = V$

So,

$$\frac{1}{2}mv^2 = eV$$

Therefore, $v = \sqrt{\frac{2eV}{m}}$

As e, m are constants so,

$$v \propto \sqrt{V}$$

So, the velocity acquired by an electron in parallel uniform electric field varies as a square root of potential difference ‘V’.

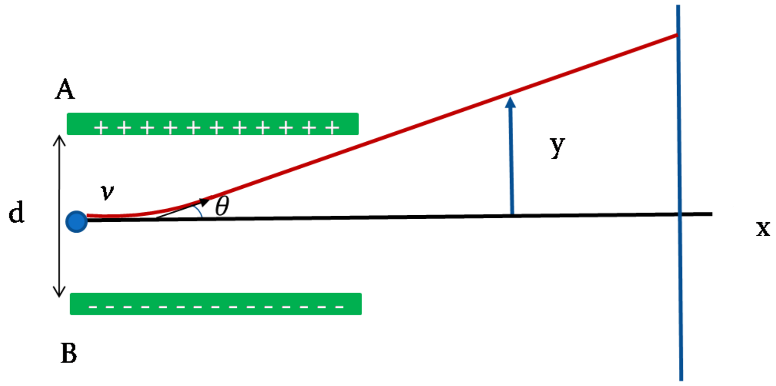
Que.1 Discuss the motion of charged particle in parallel uniform electric field. [5/6 Marks]

or

Que.2 Prove that the velocity acquired by an electron in parallel uniform electric field varies as square root of potential difference. i.e. $v \propto \sqrt{V}$ ***[5/6 Marks]***

(b) Trajectory (path) of an electron in transverse (perpendicular) uniform electric field-

When motion of electron is perpendicular to the electric field then due to electric force, there is change in the direction and magnitude of velocity of electron. Hence path of electron deviates from its straight line to parabolic nature.



-Consider an electron entering in perpendicular electric field present between the plates A and B and moving along X-axis with velocity u_x.

-Initially velocity along the Y-axis is zero.

Let, d = separation between the plates

y = vertical displacement of electron

E = magnitude of electric field (vertically downward)

The force on electron along Y axis is given as,

$$F_y = eE = ma_y$$

Acceleration along Y direction is,

$$a_y = \frac{eE}{m} \qquad \text{-----------------------(1)}$$

Let y = displacement of the electron in the field during time 't'

$$y = u_y\, t + \frac{1}{2}\, a_y\, t^2$$

$$y = \frac{1}{2}\frac{eE}{m}\, t^2 \qquad \text{--------------------- (2)}$$

Now the horizontal velocity u_x remains constant because in horizontal direction (along X-axis) there is no force acting on an electron. Thus horizontal distance travelled by an electron is given as,

$$x = u_x\, t$$

Put the value of 't' in Eq. (9)

$$y = \left(\frac{1}{2}\frac{eE}{mu_x^2}\right) x^2 \qquad \text{--------------------- (3)}$$

Here e, E, m, u_x are constants, hence we can write,

$$y = kx^2 \qquad \text{--------------------- (4)}$$

This is the equation of parabola.

Que.1 Discuss the motion of charged particle / electron in perpendicular uniform electric field. [5/6 Marks]

or

Que.2 Prove that electron follows a parabolic path when it passes through a transverse uniform electric field. i.e. $y = kx^2$ [5/6 Marks]

Motion of electron in uniform magnetic field

We know that magnetic field exerts a force on a moving charged particle 'q' which is given by,

$$\mathbf{F=q(V\times B)}$$

The magnitude of this force is given as F=q vBsinθ, where θ is angle between v and B.

(a) Trajectory (path) of an electron in parallel uniform magnetic field-

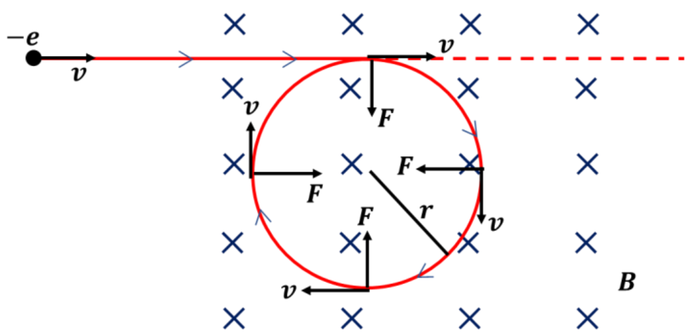
When an electron enters into the parallel magnetic field then,

$$\theta = 0$$

So, magnetic force **F** = q v B sin0 = 0

Therefore, there is no force acting on an electron in parallel magnetic field.

(b) Trajectory (path) of an electron in transverse (perpendicular) uniform magnetic field-



-When the velocity of electron (v) is perpendicular to the magnetic field (B) then angle θ = 90°.

- So the magnetic force becomes F= Bev

-Electron performs circular motion in perpendicular magnetic field.

-The centripetal force required for the circular motion is provided by magnetic force. Thus,

Magnetic force = centripetal force

$$\mathbf{Bev = \frac{mv^2}{r}}$$

Therefore, $r = \frac{mV}{eB}$ -------------------- (1)

Here e and B are constants so,

$$r \propto mV$$

The time period of revolution and frequency of revolution is given by,

Time period $=T= 2\pi r/V$

By using equation 1,

$$T= \frac{2\pi}{V} \times \frac{mV}{eB}$$

$$T= \frac{2\pi m}{eB} \qquad \text{-------------------- (2)}$$

And frequency f is given as,

$$f = 1/T = \frac{eB}{2\pi m} \qquad \text{------------------- (3)}$$

Thus the period and frequency are independent of velocity and radius of orbit.

Que.1 Discuss the motion of charged particle / electron in perpendicular uniform magnetic field. [5 Marks]

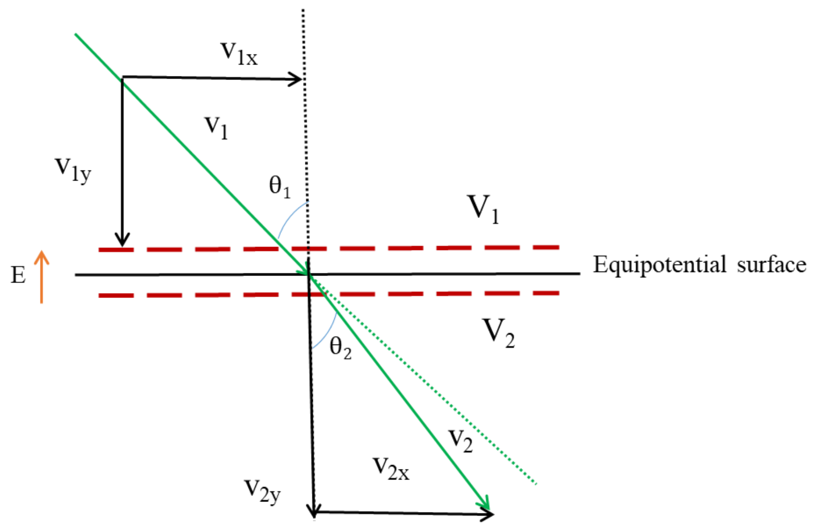
or

Que.2 Derive an expression for radius of circular path travelled by electron in perpendicular uniform

magnetic field. Also write expression for period and frequency, [5 Marks]

Bethe's law (Electron refraction) –

- Bethe's law tells us how an electron beam refracts when moving through the regions of different potentials.
- Let us consider two regions of different potentials as shown in the figure.
- Electron beam is moving through the region 1 and enters into the region 2.
- As electron beam enters into the region of potential V_2, it bends from the original path.



Let,

V_1 = Potential of region 1

V_2 = Potential of region 2 **($V_2 > V_1$)**

v_1 = velocity of electron beam in region 1

v_2 = velocity of electron beam in region 2

θ_1 = Angle of incidence

θ_2 = Angle of refraction

- As electric field is vertically upward (along Y axis), the force acing on electron will be vertically downward. Therefore Y component of velocity will change.
- There is no force acting horizontally (Along X axis), so X component of velocity will remain same.

So,

$$v_{1x} = v_{2x}$$

From figure,

$$v_{1x} = v_1 \sin\theta_1 \quad \text{and} \quad v_{2x} = v_2 \sin\theta_2$$

$$v_1 \sin\theta_1 = v_2 \sin\theta_2$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_2}{v_1} \qquad \text{............(1)}$$

We know that, kinetic energy of an electron can be written as,

$$\frac{1}{2}mv^2 = eV$$

So, Velocity can be written as,

$$v = \sqrt{\frac{2eV}{m}}$$

Therefore, $$v_1 = \sqrt{\frac{2eV_1}{m}} \quad \text{and} \quad v_2 = \sqrt{\frac{2eV_2}{m}}$$

Put in equation 1, we get

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_2}{v_1} = \frac{\sqrt{\frac{2eV_2}{m}}}{\sqrt{\frac{2eV_1}{m}}}$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_2}{v_1} = \frac{\sqrt{V_2}}{\sqrt{V_1}}$$

1) **If $V_2 > V_1$ then electron beam bends towards the normal.**
2) **If $V_2 < V_1$ then electron beam bends away from the normal.**

Que1. State and prove Bethe's law. Compare between Snell's law and Bethe's law. [5 Marks]

1. or

Que2. With the help of neat diagram, explain electron refraction at equipotential surface. [5 Marks]

Electron Lens-

- Electron lens is used to focus electron beams to a point.

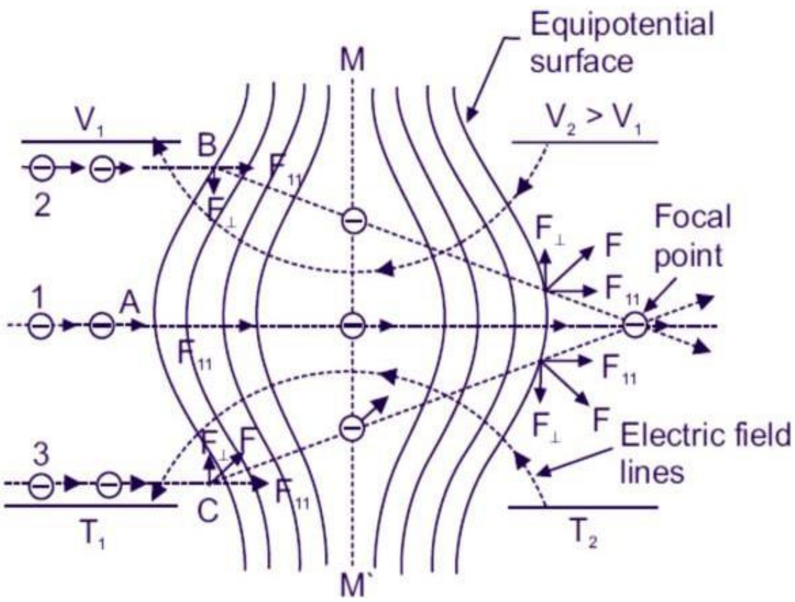
Principle (Bethe's law)-

When an electron beam travels from region of one potential to the region of another potential, it refracts.

Constuction-

- It consist of two cylindrical metal tubes T_1 and T_2 kept in front of each other and there is a gap between them.
- These tubes are kept at different potentials, so non-uniform electric field is formed in the gap between them.
- Let, V_1 = Potential of tube 1

  V_2 = Potential of tube 2 **($V_2 > V_1$)**



Working-

1) The equipotential surfaces are shown in the figure.
2) Electron beam 1- Electron beam number 1 which is moving along the axis of tubes.
3) The force acting on the electron beam is along the axis of tubes. So the beam will accelerate in forward direction.
4) Electron beam 2- Electron beam number 2 which is incident on equipotential surface in a potential V_1.
5) Due to the force acting on beam 2, it will bend. This force will have two components parallel component (F_{II}) and perpendicular component ($F\perp$).
6) Parallel component (F_{II}) pulls the beam in forward direction and perpendicular component ($F\perp$) pulls the beam in downward direction.
7) So in region of potential V_1, **action of convergence** will take place.

8) As electron beam 2, enters in to the region of potential V_2, the force **tries to diverge** the beam as shown in figure.
9) But as converging action is stronger than the diverging action, beam converges at a focal point.
10) Why converging action is stronger than the diverging action ? – Because in V_1 region speed of electron beam is **low**. So it stays in V_1 region for **longer** time and impulse due to F_{II} is **stronger** in region V_1. In V_2 region, speed of electron beam is **high** compare to speed in region V_1. So it stays in V_2 region for **short** time and impulse due to F_{II} is **smaller** in region V_2. Therefore, converging action is stronger than diverging action.

Que1. Explain principle, construction and working of electron / electrostatic lens. [5 Marks]

or

Que2. With the help of neat diagram, explain how focussing of electron beam is done using electron lens.
[5 Marks]

Cathode ray Oscilloscope (CRO)-

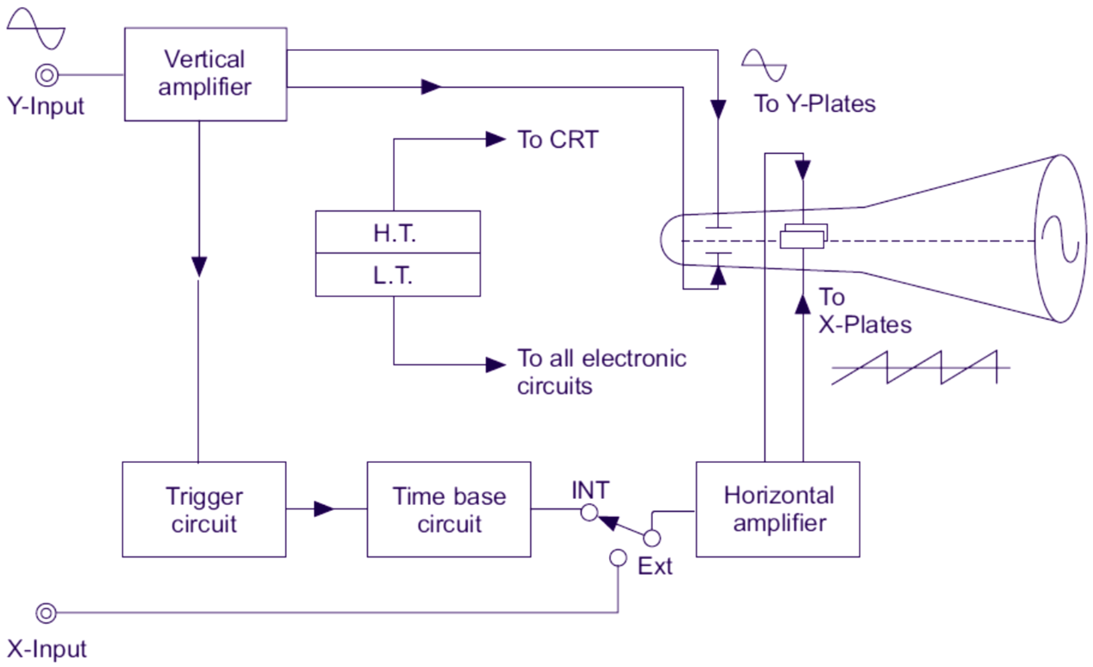


Figure: Block diagram of CRO

Any CRO contains the following seven basic sections:

1) Cathode ray tube (CRT)
2) Time base generator
3) Trigger circuits
4) Vertical circuits
5) Horizontal circuits
6) Low voltage power supply and
7) High voltage power supply

Applications of CRO:

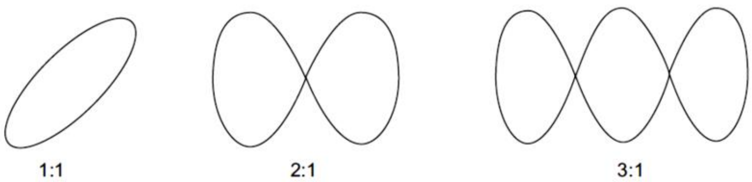
1)Determination of Amplitude of AC signal-

- The signal is fed to the vertical input of CRO.
- The number of vertical divisions is noted (peak to peak).
- Then Volts/div is noted.
- The amplitude of the signal, A, is found by the formula

$$T = (\text{Number of vertical divisions}) \times (\text{Volts/div})$$

2)Determination of unknown frequency

- The signal of unknown frequency is applied to the vertical input and a voltage of known frequency is given to the horizontal input.
- By varying the frequency of the known source, a stable loop pattern known as Lissajous pattern is obtained on the screen.



- The number of points at which the loops touch the horizontal and vertical tangents is noted.
- If nx and ny are the number of points touching the horizontal and vertical tangents respectively, then the unknown frequency is calculated from

$$f_y = \frac{n_x}{n_y} f_x$$

*Que1. **Draw block diagram of CRO and explain any one of its application in detail.***

Numericals-

1) An electron starts from rest and moves freely in an electric field of intensity 1500 V/m. Determine the force on the electron and acceleration attained by the electron.
2) A proton has an initial velocity of 2.3×10^5 m/s in the x-direction. It enters a uniform electric field of 1.5×10^4 N/C in a direction perpendicular to the field lines.
   (i) Find the time it takes for the proton to travel 0.05 m in the x-direction, and
   (ii) Find the vertical displacement of the proton after it has travelled 0.05 m in the x-direction.
3) An electron of 60 eV energy enters a perpendicular magnetic field of 0.5 T. Find velocity and radius of circular path of electron.
4) An electron of 50 eV energy enters a perpendicular magnetic field of 0.04 T. Find radius of circular path of electron.
5) An electron beam is accelerated through a potential difference of 10 kV. It is made to pass through a transverse magnetic field where it moves in a circular arc of radius 6 cm. Calculate the magnitude of magnetic field.

6) An electron is accelerated through a potential difference of 5 kV and enters a uniform magnetic field of 0.02 T acting normal to the direction of electron motion. Determine the radius of the path.
7) Electrons accelerated under a potential of 250 V enter an electric field at an angle of incidence of 50° and gets refracted through an angle of 30°. Find the potential difference between the two equipotential surfaces.
8) An electron beam enters from a region of potential 75 volts into a region of potential 100 volts, making an angle of 45° with the direction of electric field. Find the angle through which the beam refracts.

Sr	Snell's law	Bethe's law
1	Snell's law is related to refraction of light.	Bethe's law deals with refraction of electron beam.
2	$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$	$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_2}{v_1} = \frac{\sqrt{V_2}}{\sqrt{V_1}}$
3	Speed of light ray decreases when light is incident from rarer to denser medium. Light ray bends towards normal.	Speed of electron beam increases when it is incident from lower potential region to higher potential region. Electron beam bends towards normal.
4	Speed of light increases and light ray bends away from normal.	Speed of electron beam decreases and electron beam bends away from normal.

Sr	Optical Lens	Electron Lens
1	Optical lens has fixed focal length.	Focal length of electron lens can be changed by changing potentials V1 and V2.
2	Light rays are bent only at the two surfaces of a glass lens.	electron rays are bent continuously through successive equipotential surfaces
3	Converging and diverging actions are achievable using convex and concave lenses respectively.	Converging action is stronger than diverging action
4	Used in microscopes and telescopes.	Used in electron gun and electron microscopes.

Sr	**Optical Microscope**	**Electron Microscope**
1	Light is used to illuminate specimen in optical microscope	Instead of light, electron beam is used in electron microscope
2	Magnification is About 500 x to 1500 x	Magnification is About 500000 x
3	It has low resolving power.	It has high resolving power.
4	Condenser, objective and eyepiece lenses are made up of glasses.	All lenses are electromagnetic in nature.
5	Less expensive	expensive
6	Smaller and lighter, portable	Bulky and heavy set up, not portable